A New Look at $C_{n_{\beta,\mathrm{dyn}}}$

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Nomenclature

C_{n_β} = $(\partial C_n/\partial\beta)$ equilibrium
C_{l_β} = $(\partial C_l/\partial\beta)$ equilibrium
I'_x = $(I_xI_z - I_{xz}^2)/I_z$
I'_z = $(I_xI_z - I_{xz}^2)/I_x$
I_{xz} = product of inertia
I'_{xz} = $I_{xz}/(I_xI_z - I_{xz}^2)$
I_x, I_y, I_z = moments of inertia about body axes X, Y, Z, respectively
L_v = $(\partial L/\partial v)$ equilibrium
L'_v = $L_v/I'_x + I'_{xz}N_v$
N_v = $(\partial N/\partial v)$ equilibrium
N'_v = $N_v/I'_z + I'_{xz}L_v$
p = roll rate
r = yaw rate
v = Y body axes component of velocity
V_e = equilibrium velocity
Y_v = $1/m(\partial Y/\partial v)$ equilibrium
α_x = angle of attack referenced to the X body axis
ϕ = bank angle
θ_e = equilibrium pitch angle

INERTIALLY slender aircraft are known to be spin prone at high angles of attack. In trying to explain and predict spin entry conditions, various parameters have been suggested. One of these is $C_{n_{\beta,\mathrm{dyn}}}$. Moul and Paulson[1] indicated that this parameter strongly influenced the Dutch roll frequency and suggested the following form:

$$C_{n_{\beta,\mathrm{dyn}}} = C_{n_\beta}\cos\alpha - (I_z/I_x)C_{l_\beta}\sin\alpha \qquad (1)$$

For a stable condition, this parameter should be positive. Various authors have used the parameter for predicting lateral-directional instability and found the parameter to yield reasonable correlation with test data.

It should be noted that $C_{n_{\beta,\mathrm{dyn}}}$ is not an inclusive indicator of instability and should only be used in a first-cut look at stability at high angles of attack. Recent work by Kalviste[2] has shown that for equilibrium conditions with high angle of attack and sideslip, due to coupling, a static analysis should include both longitudinal and lateral equations.

This Note will show that the $C_{n_{\beta,\mathrm{dyn}}}$ criterion results from a lateral stability analysis of the perturbation equations about a symmetric equilibrium condition using only static derivatives. This Note will also develop an expression for $C_{n_{\beta,\mathrm{dyn}}}$ using general body axes. The form of this expression is somewhat more complex than Eq. (1) but reduces this equation for the special case of the body axes being principal axes. Specialization to stability axes also yields a simplified expression different from Eq. (1). Since the form of $C_{n_{\beta,\mathrm{dyn}}}$ depends on the choice of axes, care must be used to match the form of the expression used to the available aerodynamic and inertia data. Since the $C_{n_{\beta,\mathrm{dyn}}}$ criterion is developed from an analysis which includes only static derivatives, the term "dynamic" is misleading. It must be remembered that this criterion is only an indicator of instability; it provides no mathematically sound necessary or sufficient conditions.

Ignoring rate derivatives, the lateral-directional equations of motion linearized about steady, rectilinear flight are given by:

$$\begin{bmatrix}\dot v\\ \dot p\\ \dot r\\ \dot\phi\end{bmatrix} = \begin{bmatrix} Y_v & V_e\sin\alpha_x & -V_e\cos\alpha_x & g\cos\theta\\ L'_v & 0 & 0 & 0\\ N'_v & 0 & 0 & 0\\ 0 & 1 & \tan\theta_e & 0\end{bmatrix}\begin{bmatrix}v\\ p\\ r\\ \phi\end{bmatrix} \qquad (2)$$

The stability of this set of equations may be determined from the system eigenvalues. To this end, the characteristic equation is formed and is given by

$$s^3 + As^2 + Bs + C = 0 \qquad (3)$$

where a zero root has been factored out and

$$A = -Y_v \qquad (4a)$$

$$B = V_e(N'_v\cos\theta_x - L'_v\sin\alpha_x) \qquad (4b)$$

$$C = -(L'_v\cos\theta_e + N'_v\sin\theta_e)g \qquad (4c)$$

The condition under which the roots to Eq. (3) are stable are found from Routh's criterion to be

$$A > 0 \qquad (5a)$$

$$C < 0 \qquad (5b)$$

$$BA - C > 0 \qquad (5c)$$

These criteria also imply that $B > 0$.

Let us consider further the condition $B > 0$. Using Eq. (4b), we find that since V_e is always positive

$$(N'_v\cos\alpha_x - L'_v\sin\alpha_x) > 0 \qquad (6)$$

Using nondimensional stability derivatives, this expression becomes

$$(C_{n_\beta}/I'_z + I'_{xz}C_{l_\beta})\cos\alpha_x - (C_{l_\beta}/I'_x + I'_{xz}C_{n_\beta})\sin\alpha_x > 0 \qquad (7)$$

If we further specify that the body axes set used are principal axes in equality (7) reduces to

$$(C_{n_\beta}/I_{z_p})\cos\alpha_x - (C_{n_\beta}/I_{x_p})\sin\alpha_x > 0 \qquad (8)$$

or

$$C_{n_\beta}\cos\alpha_x - (I_{z_p}/I_{x_p})C_{l_\beta}\sin\alpha_x > 0 \qquad (9)$$

The left-hand side of inequality (9) is what is usually defined to be $C_{n_{\beta,\mathrm{dyn}}}$. It is clear, however, that this form is only exact when the principal body axes are used. If we specialize to the case of stability axes, we obtain

$$(C_{n_\beta}/I'_{z_s}) + I'_{xz_s}C_{l_\beta} > 0 \qquad (10)$$

Inequalities (9) and (10) can be viewed as special cases of inequality (7). The parameter $C_{n_{\beta,\mathrm{dyn}}}$ is then defined as

$$C_{n_{\beta,\mathrm{dyn}}} = (C_{n_\beta}/I'_z + I'_{xz}C_{l_\beta})\cos\alpha_x - (C_{l_\beta}/I'_x + I'_{xz}C_{n_\beta})\sin\alpha_x \qquad (11)$$

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This definition is now valied for aerodynamic and inertia data in any consistent set of body axes.

We note that the $C_{n_{\beta,\text{dyn}}}$ criterion is only one of four obtained from Routh's criterion. One of these given by inequality (5a) simply states $Y_v < 0$. The other two conditions might, in some cases, be useful to check.

Conditions for the lateral stability of an aircraft, considering only static derivatives, have been determined. From these conditions, a general body axes form of $C_{n_{\beta,\text{dyn}}}$ is defined. This form reduces to the usual form for the special case of principal axes. Care must be used that this special form not be used for other axes systems.

References

[1] Moul, M.T. and Paulson, J., "Dynamic Lateral Behavior of High Performance Aircraft," NACA RML58E16, Aug. 1958.

[2] Kalviste, Juri," Aircraft Stability Characteristics at High Angles of Attack," AGARD Symposium on Dynamic Stability Parameters, AGARD CP 235, May 1978.

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